

Homework 5
Mathematical Models

Problem 57, p. 92

Decision Variables:

x_{ij} – number of persons of type i hired for job j , $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$

Constraints

Do not hire more than available in each group:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 20$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 30$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 40$$

$$x_{41} + x_{42} + x_{43} + x_{44} \leq 20$$

Do not hire more than required for each job:

$$x_{11} + x_{21} + x_{31} + x_{41} \leq 30$$

$$x_{12} + x_{22} + x_{32} + x_{42} \leq 30$$

$$x_{13} + x_{23} + x_{33} + x_{43} \leq 40$$

$$x_{14} + x_{24} + x_{34} + x_{44} \leq 20$$

All variables $x_{ij} \geq 0$, $i, j = 1, 2, 3, 4$.

Objective function:

Maximize the number of qualified people hired:

$$\max x_{11} + x_{12} + x_{22} + x_{23} + x_{33} + x_{34} + x_{41} + x_{44}$$

Problem 20, p. 104

Part (a)

Decision variables:

s_A – start of Activity A
 t_A – end of Activity A
 s_B – start of Activity B
 t_B – end of Activity B
 s_C – start of Activity C
 t_C – end of Activity C
 s_D – start of Activity D
 t_D – end of Activity D
 s_E – start of Activity E
 t_E – end of Activity E
 s_F – start of Activity F
 t_F – end of Activity F
 s_G – start of Activity G
 t_G – end of Activity G
 T – end of project

Constraints:

$$t_A = s_A + 5$$

$$t_B = s_B + 8$$

$$t_C = s_C + 10$$

$$t_D = s_D + 5$$

$$t_E = s_E + 4$$

$$t_F = s_F + 6$$

$$t_G = s_G + 3$$

$$s_B \geq t_A$$

$$s_C \geq t_B$$

$$s_D \geq t_B$$

$$s_E \geq t_B$$

$$s_F \geq t_E$$

$$s_G \geq t_C, s_G \geq t_F$$

$$T \geq t_A, T \geq t_B, T \geq t_C, T \geq t_D, T \geq t_E, T \geq t_F, T \geq t_G$$

Objective:

$$\min T$$

Part (b)

Decision variables are the same as in (a) plus

r_A – reduction of Activity A

r_B – reduction of Activity B

r_C – reduction of Activity C

r_D – reduction of Activity D

r_E – reduction of Activity E

r_F – reduction of Activity F

r_G – reduction of Activity G

The first group of constraints change

$$t_A = s_A + 5 - r_A$$

$$t_B = s_B + 8 - r_B$$

$$t_C = s_C + 10 - r_C$$

$$t_D = s_D + 5 - r_D$$

$$t_E = s_E + 4 - r_E$$

$$t_F = s_F + 6 - r_F$$

$$t_G = s_G + 3 - r_G$$

The other constraints remain the same. Additionally, we have

$$0 \leq r_A \leq 2$$

$$0 \leq r_B \leq 3$$

$$0 \leq r_C \leq 1$$

$$0 \leq r_D \leq 2$$

$$0 \leq r_E \leq 2$$

$$0 \leq r_F \leq 3$$

$$0 \leq r_G \leq 1$$

$$T \leq 20 \quad (\text{finish in 30 days})$$

The objective changes to:

$$\min 30r_A + 15r_B + 20r_C + 40r_D + 20r_E + 30r_F + 40r_G$$

Problem 21, p. 104

Part (a)

Decision variables:

s_A – start of Activity A
 t_A – end of Activity A
 s_B – start of Activity B
 t_B – end of Activity B
 s_C – start of Activity C
 t_C – end of Activity C
 s_D – start of Activity D
 t_D – end of Activity D
 s_E – start of Activity E
 t_E – end of Activity E
 s_F – start of Activity F
 t_F – end of Activity F
 T – end of project

Constraints:

$$\begin{aligned}t_A &= s_A + 2 \\t_B &= s_B + 4 \\t_C &= s_C + 3 \\t_D &= s_D + 2 \\t_E &= s_E + 10 \\t_F &= s_F + 4 \\s_B &\geq t_A \\s_C &\geq t_B \\s_D &\geq t_B \\s_E &\geq t_C, \quad s_E \geq t_D \\s_F &\geq t_B \\T &\geq t_A, \quad T \geq t_B, \quad T \geq t_C, \quad T \geq t_D, \quad T \geq t_E, \quad T \geq t_F,\end{aligned}$$

Objective:

$$\min T$$