Modular Buildings

Problem

You manage a factory that produces small modular office buildings. You can make up to four buildings per month. If you make one or two buildings, the production cost per unit is $50,000. However, if you make three or four buildings, the production cost per unit is only $47,000. For example, making 2 buildings costs you $2 \times 50,000 = 100,000$, but making 3 buildings costs you only $3 \times 47,000 = 141,000$. Assume that you must produce a whole number of buildings each month.

You have committed to deliver the following number of buildings in the next six months: 2, 1, 5, 2, 4, 1.

You have a storage facility in which you can keep up to four finished buildings. Each month that you have any units in the storage facility, it costs you a ”flat fee” of $8,600 plus $1,300 per unit stored. If you have no inventory stored in the facility, it does not cost you anything. You currently do not have any inventory stored in the facility.

Solution

Decision variables (for each month $t = 1, \ldots, 6$):

\[
\begin{align*}
    z_t &= \begin{cases} 
    1 & \text{produce full-cost buildings in month } t \\
    0 & \text{do not} 
    \end{cases} \\
    w_t &= \begin{cases} 
    1 & \text{produce reduced-cost buildings in month } t \\
    0 & \text{do not} 
    \end{cases} \\
    x_t &= \text{number of full-cost buildings made in month } t \\
    y_t &= \text{number of reduced-cost buildings made in month } t \\
    o_t &= \begin{cases} 
    1 & \text{open inventory in month } t \\
    0 & \text{do not} 
    \end{cases} \\
    I_t &= \text{number of buildings in inventory in month } t
\end{align*}
\]

Constraints:

Either-or:

\[ z_t + w_t = 1, \quad t = 1, \ldots, 6, \]

Logical lower and upper bounds

\[
\begin{align*}
    0 &\leq x_t \leq 2z_t, \quad t = 1, \ldots, 6 \\
    3w_t &\leq y_t \leq 3w_t, \quad t = 1, \ldots, 6 \\
    0 &\leq I_t \leq 4o_t, \quad t = 1, \ldots, 6
\end{align*}
\]
Inventory equation (with $I_0 = 0$ and $d_t$ representing the demand in month $t$):

$$I_t = I_{t-1} + x_t + y_t - d_t, \quad t = 1, \ldots, 6$$

$z_t, w_t, o_t$ - binary, $x_t, y_t$ - integer.

*Objective*:

$$\min \sum_{t=1}^{6} \left( 50000x_t + 47000y_t + 8600o_t + 1300I_t \right)$$